Coastlab12, 2012, Ghent, Belgium. Book of Abstracts of the Fourth International Conference on the Application of Physical Modelling to Port and Coastal Protection. ISBN xxx-xxxx-xx-x

COMPLEX REFLECTION COEFFICIENTS APPLIED TO STEEP SLOPING STRUCTURES

FRITZ BÜSCHING

Bielefeld University of Applied Sciences, Germany,
Dießelhorststr. 1, 38116 Braunschweig, buesching@hydromech.de

1. Introduction

The total surf zone processes at steep slopes can be looked upon as a combination of reflection, transmission and dissipation phenomena. Such a result from numerous model investigations in the wave tank of Bielefeld University of Applied Sciences appears to be an analogue to other kinds of waves, especially to electromagnetic waves at uniform planar interfaces.

Accordingly in the course of the dissipative wave breaking process, a wave pulse of transmission evolves from the initial incident wave at the landward side, while a reflected wave is produced at the seaward side at the same time. The wave pulse of transmission is characterized by a wave height $H_t < H_i$ and phase velocity $c_t < c_i$ and the reflected wave height is $H_r < H_i$.

In this process it is essential that due to the conservation of momentum, the positive water level deflection of the transmitted wave pulse postulates locally a negative water level deflection of the reflecting wave. Hence, the superimposition of incident and reflected waves results in a partially standing wave comprising of a phase jump. The partial clapotis node close to the point IP, where the structure front face intersects the still water level, can be regarded as a center of rotation, around which the water level deflections of the washing movement (runup – rundown) and those of the partial standing wave occur in opposite phases, see Figure 1.

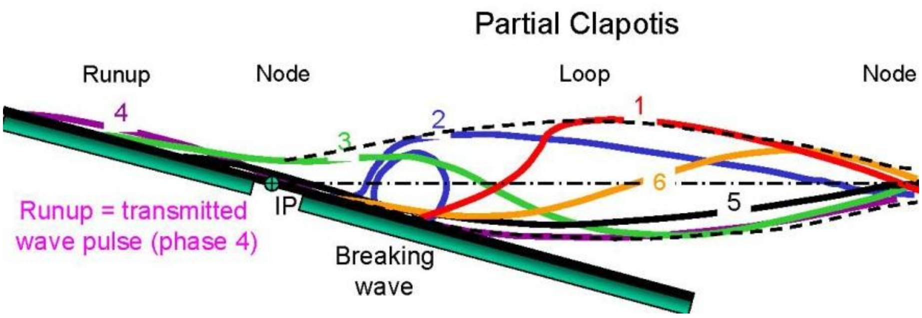

Figure 1. Six phases of a plunging breaker at the condition of a phase jump, caused by partial reflection and by the transmitted pulse of wave run up. In phases 3 and 4, there are opposite water level movements at both sides of the imperfect Clapotis node nearly coinciding with IP.

As the reflected wave differs from the incident wave both in wave height and phase, at superimposition a partial standing wave is formed, which is characterized not only by the difference ΔH but also by $\Delta\varphi$. Hence, the reflection coefficient can be defined as a complex quantity Γ, which is similar to the respective definition in telecommunications of a discontinuity in a transmission line:

$$\Gamma = C_r e^{i\Delta\varphi} \qquad [1]$$

where the magnitude is $C_r = H_r/H_i$ and the phase $\Delta\varphi$ is the phase difference between the incident and the reflected wave.

The expression of the total wave field in the complex notation with reference to the amplitude A (= H/2) is:

$$y(x,t) = Ae^{i(\omega t - kx)} + C_r Ae^{i(\omega t + kx + \Delta\varphi)} = (e^{-ikx} + C_r e^{i\Delta\varphi} e^{ikx})Ae^{i\omega t} = (e^{-ikx} + \Gamma\, e^{ikx})Ae^{i(\omega t)} \qquad [2]$$

where the angular frequency $\omega = 2\pi/T$ and the wave number $k = 2\pi/L$.

2. Methods

Magnitude and phase of the complex reflection coefficient can be determined on the basis of the wave field (envelope of water level deflections) measured experimentally seaward of the point IP, where the structure front face intersects the still water level.

The magnitude C_r can be determined by applying Healy´s formula:

$$C_r = \frac{H_{max} - H_{min}}{H_{max} + H_{min}} \quad \text{where} \quad H_{max} = H_i + H_r \quad \text{and} \quad H_{min} = H_i - H_r \qquad [3]$$

and the phase difference $\Delta\varphi$ results from

$$\Delta\varphi[°] = 360(1 - 2\eta_{max}/L) \qquad [4]$$

where η_{max} is the distance between the point of reflection IP and the nearest loop (anti node) and L is the wave length.

Analyzing irregular waves (and their Fourier-components) instead of the wave field of water level deflections, in principal the square values of the water level deflections can be used advantageously. As an example this was done for 5 partial waves representing narrow frequency sections of the spectrum´s core.
Two different types of revetments inclined 1 : 2 had been tested synchronously: Smooth slope versus "Hollow Cubes".

3. Results and discussion

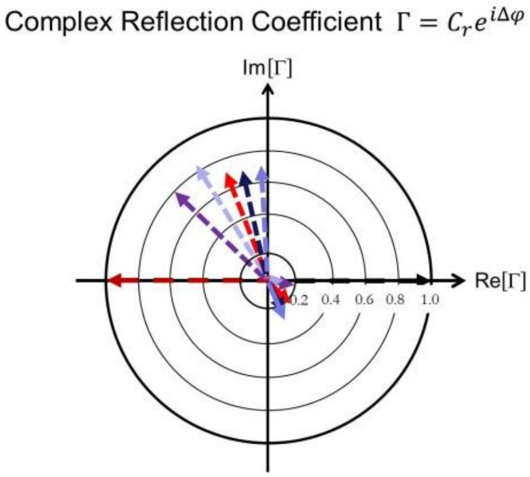

Figure 2. Phasor diagram (complex number plane)

Figure 2 shows the phasor diagram of complex reflection coefficients belonging to the 5 partial waves each at the two different revetments inclined 1 : 2. Additionally the phasors of the 2 theoretical total reflection cases - positive ($C_r = 1$, $\Delta\varphi = 0°$) and negative ($C_r = 1$, $\Delta\varphi = 180°$) are to be seen. The 5 big phasors in the second quadrant belong to the smooth slope and the 5 small ones in the fourth quadrant to Hollow Cubes.
It can be stated that the big difference between the two kinds of revetments is not only expressed by the respective multiple of the magnitudes of the reflection coefficients but also by the big phase distances $90° < \Delta\Delta\varphi < 180°$ between the phasors. Moreover magnitudes and phases both clearly depend on frequency.
A prominent consequence of defining a complex reflection coefficient Γ can be seen in its feature distinguishing between positive and negative reflection. In this context, the above theoretical limiting cases of different kinds of total reflection give the actual definitional elements. Hence, the case of positive theoretical retro-reflection from an ideal smooth vertical wall without a phase jump is accompanied by an equivalent negative theoretical case of reflection with a phase jump of $\Delta\varphi = 180°$ - most probably from an ideal smooth inclined wall.
At positive total reflection a (critical) loop exists at the reflection point, because the wave crest is reflected by a wave crest. Contrary at negative total reflection there is a node precisely at the point of reflection, because a wave crest is reflected by a wave trough and vice versa.
At partial standing waves - in addition to the Iribarren-number - the real part of the reflection coefficient Re[Γ] as well as its imaginary part Im[Γ] might be the key factors deciding on the type of breaker to be produced, because the actual phase shift $\Delta\varphi$ controls the positioning of the partial standing wave with reference to the reflection point IP. Results on additional slope angles and frequencies will be presented in the forthcoming paper.

References

Büsching, Fritz 2012. 'Komplexe Reflexionskoeffizienten für Wasserwellen', *Die Küste*, H.79 (2012) in printing.